CONCURSUL INTERJUDEŢEAN DE MATEMATICĂ

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BAREM

CLASA a XI a

1. Determinaţi $X \in M_2(\mathbb{R})$ dacă $X \cdot \begin{pmatrix} 1 & 2 \\ 1 & 5 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$.

Prof. Dan Negulescu, Brăila

***Soluţie*:** Notăm $Y = X \cdot \begin{pmatrix} 1 & 2 \\ 1 & 5 \end{pmatrix}$ atunci ecuaţia devine

$Y^2 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 0 & -6 \\ 0 & 6 \end{pmatrix}$; $(\det Y)^2 = 0 \Rightarrow \det Y = 0.\ (1p)$ Notăm $Y = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, ad - bc = 0 \Rightarrow$

$Y^2 = (a+d)Y \Rightarrow (a+d)a = 0; (a+d)b = -6; (a+d)c = 0, (a+d)d = 6 \Rightarrow a = 0, d^2 = 6, (2p)$

Pentru $d = \sqrt{6} \Rightarrow X = \frac{\sqrt{6}}{3}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.(2p)$

Pentru $d = \sqrt{6} \Rightarrow X = \frac{\sqrt{6}}{3}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}.(2p)$

2. Fie şirul $(a_n)_{n\in\mathbb{N}}$ pentru care $a_{n+1} = 3^n - 2015a_n$, oricare ar fi $n \in \mathbb{N}$. Aflaţi a_0 pentru care şirul $(a_n)_{n\in\mathbb{N}}$ este strict crescător şi apoi calculaţi $\lim\limits_{n\to\infty} \frac{a_n}{n!}$.

Student, Andreea Maria Popa, Universitatea Bucureşti

***Soluţie*:**

$$\left.\begin{array}{l} a_{n+1} = -2015a_n + 3^n \\ (-2015)a_n = (-2015)^2 a_{n-1} + 3^{n-1}(-2015) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (-2015)^n a_1 = (-2015)^{n+1} a_0 + 3^0(-2015)^n \end{array}\right\} \Rightarrow a_{n+1} = (-2015)^{n+1} a_0 + 3^n + 3^{n-1}(-2015) + \ldots + 3^0(-2015)^n =$$

$$= (-2015)^{n+1} a_0 + 3^n\left[1 + \left(\frac{-2015}{3}\right)^1 + \left(\frac{-2015}{3}\right)^2 + \ldots + \left(\frac{-2015}{3}\right)^n\right] = (-2015)^{n+1} a_0 + 3^n \frac{1 - \left(\frac{-2015}{3}\right)^{n+1}}{1 + \frac{2015}{3}} =$$

$$=(-2015)^{n+1}a_0+\frac{3^{n+1}}{2018}\left[1-\left(\frac{-2015}{3}\right)^{n+1}\right]=\frac{3^{n+1}}{2018}-\frac{1}{2018}(-2015)^{n+1}+(-2015)^{n+1}a_0=\frac{3^{n+1}}{2018}+$$

$$+(-2015)^{n+1}\left(a_0-\frac{1}{2018}\right)\Rightarrow a_n=\frac{3^n}{2018}+(-2015)^n\left(a_0-\frac{1}{2018}\right)\ (3p)\text{strict crescător}\Rightarrow a_0=\frac{1}{2018}(1p)\Rightarrow$$

$$\Rightarrow a_n=\frac{3^n}{2018}\Rightarrow\lim_{n\to\infty}\frac{a_n}{n!}=0.(3p)\ \text{justificare!}$$

3. Fie şirul de numere reale $(x_n)_{n\geq 0}$, cu proprietatea $x_{n+k}+x_{n-k}=x_{5n}$, oricare ar fi $n,k\in\mathbb{N}$ şi $n\geq k$. Calculaţi A^n, unde

$$A=\begin{pmatrix}1+x_{2016} & 0 & 0\\ 2 & 1+x_{2016} & 0\\ 3 & 4 & 1+x_{2016}\end{pmatrix}.$$

Prof. Carmen şi Viorel Botea, Brăila

***Soluţie*:**

$k=n=0\Rightarrow x_0=0,\ k=n\Rightarrow x_{2n}=x_{5n}\Rightarrow x_{4n}=x_{2\cdot(2n)}=x_{5\cdot(2n)}=x_{2\cdot(5n)}=x_{5\cdot(5n)}=x_{25n}$ pentru orice $n\in\mathbb{N}$. Pentru $n\to 5n$ şi $k=n\Rightarrow x_{6n}+x_{4n}=x_{25n}\Rightarrow x_{6n}=0\Rightarrow x_{2016}=0.(3p)$

$$A=\begin{pmatrix}1&0&0\\2&1&1\\3&4&1\end{pmatrix}=I_3+\begin{pmatrix}0&0&0\\2&0&0\\3&4&0\end{pmatrix}=I_3+B\Rightarrow A^n=\sum_{k=0}^{n}C_n^kB^k\Rightarrow B^2=B\cdot B=\begin{pmatrix}0&0&0\\0&0&0\\8&0&0\end{pmatrix}\Rightarrow B^3=O_3\Rightarrow$$

$$A^n=C_n^0I_3+C_n^1B+C_n^2B^2=\begin{pmatrix}1&0&0\\2n&1&0\\4n^2-n&4n&1\end{pmatrix}.(4p)$$